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Well-posedness Techniques in the Qualitative Analysis for a class of Equilibrium problems

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Abstract
The main goal of this paper is to establish some well-posedness results for a nonstandard equilibrium problem (for short EP_N) and for optimization problem involving α-monotone bifunction, whose solution is sought in a subset K of a real reflexive Banach space X. Moreover, we establish some metric characterizations of well-posedness for a nonstandard equilibrium problems and for an optimization problem.

1.Introduction

It is well-posedness plays a crucial role in the stability analysis and numerical methods for optimization theory and nonlinear operator equation. The concept of well-posedness of unconstrained and constrained scalar optimization problems was first introduced and studied by Levintin and Polyak [14] and by Tykhonov [19], respectively, which has been known as the Levitin-Polyak and Tykhonov well-posedness, respectively. There are in the literature a many of papers dealing with a generalization of Tykhonov well-posedness relating with optimization problems with more than one solution. This requires the existence and the convergence of subsequence of every minimizing sequence towards a solution. For more details, we refer readers to (see, e.g., [2, 4, 5, 11, 21]). Fang et al. [7] investigated the well-posedness of equilibrium problems; Kimura et al. [12] studied the parametric well-posedness for vector equilibrium problems; Bianchi et al. [3] introduced and studied two types of well-posedness for vector equilibrium problems; SJ and MH [18] investigated the Levitin-Polyak well-posedness of vector equilibrium problems, with variable domination structures, Salamon [17] analyzed the Hadamard well-posedness of parametric vector equilibrium problems, Peng et al. [16] investigated several types of Levitin-Polyak well-posedness of generalized vector equilibrium problems. Long et al. [15] and Zaslavski [20] introduced the notions of generalized Levitin-Polyak well-posedness forexplicit constrained EPs and generic well-posedness for Eps, respectively. Most of these works considered the perturbation of the parameters in the vector-valued case.

The aim of this work is to give a new contribution in this area. In particular, we establish some concepts of well-posedness by parametric for new class of equilibrium problems EP_N and for optimization problems with perturbations which includes in special case the classical equilibrium problems.

We further prove that the well-posedness of generalized equilibrium problems is equivalent to the existence and uniqueness of its solution. The distinguishing feature of our work lies in "ask F not to be monotone bifunction (as in most papers dealing with equilibrium problems in well-posedness), but to be $\alpha-\text{monotone}$ (which is rather a weak condition compared to monotonicity)".

In order to achieve the aim, the study is divided into the following sections. In section 2, we recall to some definitions and results that need to present our main results. In section 3, we establish and generalize the concept of well-posedness for equilibrium problems to generalized equilibrium problems (EP_N). And derive some metric characterizations of well-posedness. In section 4, we present a new concept of well-posedness for optimization problems with constrains described by parametric generalized equilibrium problems. In the last section, we include some concluding remarks.

2. Preliminaries

In this paper, unless stated otherwise, it is assumed that E and X are two real reflexive Banach spaces and let K be a nonempty convex, closed subset of a Banach space X and X^* it is the topological dual space, while $\|.\|$ denote the norm in X^*. And that $A : K \longrightarrow E^*$ be a nonlinear mapping, where K is a nonempty subset of X.

For the convenience of the reader, we recall some important definitions and useful results that need to be imposed in order to prove our main results. Let us consider the following a nonstandard equilibrium problem [9] (for short EP_N) is to find $x \in K$ such that

$$F(x,y)+\Psi(x,y)+\langle Ax, y-x\rangle \geq 0 \ \ \forall y \in K, \qquad (2.1)$$

in which $F, \Psi : K \times K \longrightarrow \mathbb{R}$ are two bifunctions and F is a nonstandard equilibrium problem (EP_N) with $F(x,x)=\Psi(x,x)=0 \quad \forall x \in K$.

In what follows, we introduce the formulation of optimization problems with equilibrium constraint.

Let $h : T \times K \longrightarrow \mathbb{R}$ and $F : T \times K \times K \longrightarrow \mathbb{R}$ be two functions, in which $T \subset E$ is a nonempty set. The optimization problem with generalized equilibrium constraint (denoted by (OPNPEC)) is formulated as follows:

$$\min h(t,u) \ \ \text{s.t} \ (t,u) \in T \times K \text{ and } \ u \in S(t),$$

where $S(t)$ is the solution set of the parametric generalized equilibrium problem ($EP_N(t)$) defined by $u \in S(t)$ if and only if

$$F(t,u,v)+\Psi(u,v)+\langle Au, v-u\rangle \geq 0 \ \ (\forall v \in K). \qquad (2.2)$$

Instead of writing $\{EP_N(t) : \forall t \in T\}$ for the family of a nonstandard equilibrium problems i.e., the parametric problem, we will simply write (EP_N) in the sequel.

Throughout this paper, the *inner product* in Hilbert space H, denoted by $\langle .,. \rangle$, is defined as $\langle .,. \rangle : H \times H \longrightarrow \mathbb{C}$ is inner product in H, if it satisfies the following conditions. Let u, v and w be vectors and r, m be two scalars, then:

1. $\langle u, rv+mw\rangle = \bar{r}\langle u,v\rangle + \bar{m}\langle v,w\rangle$.
2. $\langle u,u\rangle \geq 0$ and equal if and only if u = 0 for all $u \in H$.
3. $\overline{\langle v,u\rangle} = \langle u,v\rangle$ for all $u, v \in H$.

In order to highlight the generality of the problem (EP_N), we recall below some special cases, as below:

(i) If $\Psi \equiv A \equiv 0$, then problem (2.2) is reduces to the parametric equilibrium problem (for short, $EP(t)$), in fact, finding $x \in K$ such that $F(t,x,y) \geq 0 \ \ \forall y \in K$ (see [1]).

(ii) If $\Psi \equiv A \equiv 0$ and $F(t,x,y) = -h(t,x,x-y) \ \ \forall y \in K$, then problem (2.2) is reduces to the param-etric quasivariational inequality (for shot, QVI(t)) see [22].

(iii) $A \equiv 0$ and $\Psi(x,y) = \Psi(x) - \Psi(y)\ \forall y \in K$ and $F(t,x,y) = \langle F(x), y - x\rangle$, then problem (2.2) is reduces to the mixed variational inequalities (for short, (MVI)) see [6].

(iv) If $A \equiv 0$, then problem (2.2) reduces to the generalized equilibrium problem (for shot, (EP_N)) see [8].

In recent year, some of the authors have proposed many essential generalizations of monotonicity. We shall use a kind of generalized monotonicity, so called $\alpha - \mathrm{monotone}$ bifunction.

Definition 2.1. [10] Let $\alpha : K \times K \longrightarrow \mathbb{R}$ be a real-valued function. A bifunction $F : K \times K \longrightarrow \mathbb{R}$ is called $\alpha - \mathrm{monotone}$ if

$$F(x,y) + F(y,x) + \alpha(x,y) \le 0 \ \ (\forall x, y \in K). \tag{2.3}$$

Definition 2.2. [13] A real-valued function G, defined on a convex subset K of X, is said to be hemicontinuous, if

$$\lim_{t\to 0^+} G(tx + (1-t)y) = G(y) \quad (\forall x, y \in K). \tag{2.4}$$

Here, we consider the following assumptions in [10], $\forall r \in [0,1]$

Definition 2.3. [13] Let X be a Banach space. A mapping $\Lambda : X \longrightarrow \mathbb{R}$ is said to be

(i) lower semicontinuous (for short, (l.s.c)) at $x_0 \in X$, if

(ii) upper semicontinuous (for short, (u.s.c)) at $x_0 \in X$, if

for any sequence x_n of X such that $x_n \longrightarrow x_0$.

Let us recall that the concepts of a noncompactness measure and Hausdorff metric.

Definition 2.4. [13] Let M, N be nonempty subsets of X. The Hausdorff metric $H(.,.)$ between N and M is defined by

$$H(N,M) = \max\{e(N,M), e(M,N)\},$$

where $e(N,M) = \sup_{a\in N} d(a,M) = \inf_{b\in M}\|a-b\|$. Let $\{N_r\}$ be a sequence of nonempty subsets of X. We say that N_n converges to N in the sense of Hausdorff metric if $H(N_r, N) \to 0$. It is easy to see that $e(N_r, N) \to 0$ if and only if $d(a_r, N) \to 0$ for all selection $a_r \in N_r$. For more details on this topic, we refer the readers to [15].

Definition 2.5. [13] Assume that A is a nonempty subset of X. The measure of a noncompactness β of the set A is defined by

where diam means the diameter of a set.

Recall that the diameter of a subset A, is defined as diam (A) = sup $\{\|a-b\| : a, b \in A\}$, provided the set A is nonempty.

We end this section with theorem that will play a key role in the proof of our main results.

Theorem 2.6. [9] Assume that K is a nonempty subset of a real reflexive Banach space X, $F : K \times K \longrightarrow \mathbb{R}$ is $\alpha - \mathrm{monotone}$ bifunction, hemicontinuous in the first argument and convex in the second argument. Let $\Psi, \alpha : K \times K \longrightarrow \mathbb{R}$ be convex in the second argument, and that $A : K \longrightarrow X^*$ be arbitrary nonlinear operator. Then a nonstandard equilibrium problem (EP_N) is equivalent to the following problem: Find $x \in K$ such that

$$\langle Ax, x-y\rangle + F(y,x) + \alpha(x,y) \leq \Psi(x,y)\ (\forall y \in K). \quad (2.9)$$

3. Well-posed of (EP_N) with metric characterizations

In this section we establish some concepts of well-posed of generalized equilibrium problem (EP_N). To start our analysis. Through the results of this section, we give some conditions under which the equilibrium problem is strongly well-posed in the generalized sense.

Definition 3.1. A sequence $\{(t_n, x_n)\} \subset T \times K$ is said to be an approximating sequence for (EP_N) if there exists a nonnegative sequence $\{\epsilon_n\}$ with $\{\epsilon_n\} \longrightarrow 0$ as $n \longrightarrow \infty$ such that

$$F(t, x_n, y) + \Psi(x_n, y) + \langle Ax_n, y - x_n\rangle \geq -\epsilon_n \|y - x_n\|\ \forall n \in \mathbb{N}, y \in k. \quad (3.1)$$

Definition 3.2. The problem (EP_N) is said to be strongly well-posed (resp., strongly well-posed in the $\{x_n\}$ with $x_n \longrightarrow x$, every generalized sense) if (EP_N) has a unique solution x, and for every sequence approximation sequence for (EP_N) converges strongly to the unique solution (resp., if (EP_N) has a nonempty solution set $S(t)$, and every approximate solution sequence has a subsequence which strongly to some point of $S(t)$).

In what follows, we shall establish some characterization of well-posedness for (EP_N).

For any $\epsilon > 0$ we define two sets:

and

$\Lambda(\epsilon$

Lemma 3.3. Suppose that K is a nonempty convex, closed subset of a real reflexive Banach space X. Let $F : T \times K \times K \longrightarrow \mathbb{R}$, $A : K \longrightarrow X^*$ and Ψ, $\alpha : K \times K \longrightarrow \mathbb{R}$ be four functions. Moreover, the following conditions hold:

(i) $F(t, x, x) = 0\ \ \forall t \in T, x \in K$,
(ii) $F(t, ., .)$ is $\alpha -$ monotone bifunction and hemicontinuous $\forall t \in K$,
(iii) $F(t, x, .)$ is convex $\forall t \in T, x \in K$,
(iv) $\Psi(x, .)$ and $\alpha(x, .)$ are convex $\forall x \in K$.

Then $\Gamma(\epsilon) = \Lambda(\epsilon)$ for all $\epsilon > 0$.

Proof. Suppose that $(t, x) \in \Gamma(\epsilon)$. There exists $(t, x) \in T \times K$ such that

Since $F(t, ., .)$ is $\alpha -$ monotone bifunction

then

$$\begin{aligned} F(t, y, x) + \alpha(x, y) &\leq -F(t, x, y) \\ &\leq \Psi(x, y) + \langle Ax, y - x\rangle + \epsilon\|y - x\|. \end{aligned} \quad (3.3)$$

So $(t, x) \in \Lambda(\epsilon)$. Therefore, $\Gamma(\epsilon) \subseteq \Lambda(\epsilon)$. Conversely, assume that $(t, x) \in \Lambda(\epsilon)$ and fix $y \in K$.

Letting $x_\lambda = x - \lambda(x - y)$, $\lambda \in]0, 1[$. Then $x_\lambda \in K$ since K is a convex. So

$$= \quad \lambda[\ \langle Ax, y - x\rangle + \epsilon\|y - x\|\ ] - \alpha(x, x_\lambda) + \Psi(x, x_\lambda). \quad (3.4)$$

Taking into account $F(t, x, .)$ is convex

So,

Since $\Psi(x, .)$ and $\alpha\ (x, .)$ are convex $\forall x \in k$, *then*

From (3.4), (3.5), (3.6) and (3.7), one can get

Since $F(t,.,.)$ is hemicontinuous, then

$\lambda[-F(t,x,y)$

$+(1-\lambda)[\Psi(x,x)+\alpha(x,x)].$

So,

$F(t,x,y)+\Psi$

From (2.5) and (2.6), one can obtain

Hence, $(t,x)\in\Gamma(\epsilon)$. Therefore, $\Gamma(\epsilon)=\Lambda(\epsilon)$ for all $\epsilon>0$.

Lemma 3.4. Let K be a nonempty convex, closed subset of a real reflexive Banach space X. Suppose that $F: T\times K\times K\longrightarrow\mathbb{R}$ and $\Psi,\alpha: T\times K\times K\longrightarrow\mathbb{R}$ are three functions and $A: K\longrightarrow X^*$ is arbitrary nonlinear operator. Satisfy in the following conditions:

(i) $x\longmapsto\langle Ax,y-x\rangle$ is u.s.c on K with respect to $\text{weak}^*-\text{topology of } X^*$,

(ii) $F(.,x,.)$ and $\alpha(.,y)$ are l.s.c $\forall y\in K$,

(iii) $\Psi(.,y)$ is u.s.c $\forall y\in K$.

Then $\overline{\Lambda(\epsilon)}=\Lambda(\epsilon)$ in $T\times K$ for any $\epsilon>0$.

Proof. Since the fact that $\Lambda(\epsilon)\subseteq\overline{\Lambda(\epsilon)}$ holds always , then it is enough to show that $\overline{\Lambda(\epsilon)}\subseteq\Lambda(\epsilon)$. Assume that $(t,x)\in\overline{\Lambda(\epsilon)}$ then there exists $\{(t_n,x_n)\}\subset\Lambda(\epsilon)$ which converge sequence to (t,x) in $T\times K$.

So,

$F(t_n,y,x_n)+\alpha(x_n,y)\le\Psi(x_n,y)+\langle Ax_n,y-x_n\rangle+\epsilon\|y-x_n\|$ $(\forall y\in K)$.

From conditions (i-iii), then

$F(t,y,x)+\alpha$

$\le\Psi(x,y)+\langle Ax,y-x\rangle+\epsilon\|y-x\|,$

which implies that $(t,x)\in\Lambda(\epsilon)$. Therefore, $\overline{\Lambda(\epsilon)}=\Lambda(\epsilon)$ in $T\times K$ for any $\epsilon>0$.

The first main result of this paper is given by the following theorem.

Theorem 3.5. Assume that K is a nonempty convex, closed subset of a real reflexive Banach space X. Let $F: T\times K\times K\longrightarrow\mathbb{R}$, $A: K\longrightarrow X^*$ and $\Psi,\alpha: K\times K\longrightarrow\mathbb{R}$ be four functions. If (EP_N) is strongly well-posed. Then

Moreover, if the following assumptions hold:

(i) $x\longmapsto\langle Ax,y-x\rangle$ is u.s.c on K with respect to weak^*-topology of x^*,

(ii) $F(t,x,x)=0$, $\forall t\in T, x\in K$,

(iii) $F(.,x,.)$ is l.s.c and convex $\forall x\in K$,

(iv) $F(t,.,.)$ is $\alpha-$ monotone bifunction, hemicontinuous, $\forall t\in T$,

(v) $\alpha(.,y)$ and $\Psi(.,y)$ are u.s.c $\forall x\in K$,

(vi) $\Psi(x,.)$ and $\alpha(x,.)$ are convex $\forall x\in K$,

(vii) $\Psi(.,y)$ is u.s.c $\forall y\in K$.

Then the converse holds.

Proof. Assume that (EP_N) is strongly well-posed. Then, (EP_N) admite a unique solution $(t, x) \in T \times K$, i.e.,

Clearly, $\Gamma(\epsilon) \neq \emptyset$ for any $\epsilon > 0$. By the contradiction, assume that $\lim_{\epsilon \to 0} \operatorname{diam}\left(\Gamma(\epsilon)\right) > p > 0$.
for some nonnegative sequence $\{\epsilon_n\}$, We could find two sequences $\{(t_n, x_n)\}$ and $\{(t_n, y_n)\}$ satisfy $(t_n, x_n) \in \Gamma(\epsilon)$, $(t_n, y_n) \in \Gamma(\epsilon)$, and

Since $\{(t_n, x_n)\}$ and $\{(t_n, y_n)\}$ are approximating sequence for (EP_N). By the well-posedness of (EP_N) they have to converge strongly to the unique solution of (EP_N) a contradiction to (3.10).
Conversely, suppose that condition (3.9) holds. Let $\{(t_n, x_n)\}$ be an approximating sequence for (EP_N) so, there exists a nonnegative sequence $\{\epsilon_n\}$ with $\{\epsilon_n\} \longrightarrow 0$ as $n \longrightarrow \infty$ such that

This yields that $\{(t_n, x_n)\} \in \Gamma(\epsilon_n)$. It follows form (3.9) that $\{(t_n, x_n)\}$ is a Cauchy sequence and so it converges strongly to a point $(t, x) \in T \times K$. It follows from (3.11), $\alpha -$ monotonicity and because $\alpha(., y)$, $F(., y, .)$ are l.s.c and $\Psi(., y)$ is u.s.c, that

The fact together with Theorem 2.6. (t, x) solves (EP_N). To complete the proof, it is sufficient to show that (EP_N) has a unique solution. If (EP_N) has two distinct solutions (x_1, u_1) and (x_2, u_2), it is easily seen that (x_1, u_1), $(x_2, u_2) \in \Gamma(\epsilon_n)$ for all $\epsilon > 0$. It is follows that

This yields $(x_1, u_1) = (x_2, u_2)$ and so (EP_N) has a unique solution.

Remark 3.6. Assume that diameter of Γ does not tend to zero, if (EP_N) has more than one solution. In the next result we consider the Kuratowski noncompactness measure of approximating solution set instead of the diameter.

Theorem 3.7. Assume that T and K are nonempty, closed and convex subsets of real reflexive Banach spaces E and X respectively. If (EP_N) is strongly well-posed in the generalized sense. Then

$$\forall \epsilon > 0, \lim_{\epsilon \to 0} \beta\left(\Gamma(\epsilon)\right) = 0, \text{ where } \Gamma(\epsilon) \neq \emptyset. \tag{3.12}$$

Moreover, if the following assumptions hold:

(i) $x \longmapsto \langle Ax, y - x\rangle$ is u.s.c on K with respect to weak* $-$topology of X^*,
(ii) $F(t, x, x) = 0, \forall t \in T, x \in K$,
(iii) $F(., x, .)$ is l.s.c and convex $\forall x \in K$,
(iv) $F(t, ., .)$ is $\alpha -$ monotone bifunction, hemicontinuous, $\forall t \in T$,
(v) $\alpha(., y)$ and $\Psi(., y)$ are u.s.c $\forall x \in K$,
(vi) $\Psi(x, .)$ and $\alpha(x, .)$ are convex $\forall x \in K$,
(vii) $\Psi(., y)$ is u.s.c $\forall y \in K$.

Then the converse holds.

Proof. Let (EP_N) be strongly well-posed in the generalized sense. Then the solution set S of (EP_N) is a nonempty. This indicates that, for any $\epsilon > 0, \Gamma(\epsilon) \neq \emptyset$ because $S \subset \Gamma(\epsilon)$. Moreover, we claim here that the solution set S of (EP_N) is compact. Indeed, for any sequence $\{(t_n, x_n)\}$ in S. $\{(t_n, x_n)\}$ is an approximating sequence for (EP_N). Thus there exists a converging subsequence to some point of S. This implies that S is compact. Now, we show that $\lim_{\epsilon \to 0} \beta\left(\Gamma(\epsilon)\right) \to 0$. It follows from $S \subset \Gamma(\epsilon)$ that

Since the solution set S is compact, then

where $\beta(S) = 0$, since S is compact. To prove $\lim_{\epsilon\to 0} \beta\big(\Gamma(\epsilon)\big) = 0$. It is sufficient to show that $e(\Gamma(\epsilon), S) \to 0$ as $\epsilon \to 0$. If not, there exists a constant $c > 0$ and $\{\epsilon_n\} \to 0$, and $\{(t_n, x_n)\} \subset \Gamma(\epsilon_n)$ in which

where $B_{\frac{c}{2}}(0)$ is an open ball with center 0 and radius $\frac{c}{2}$. However, $\{(t_n, x_n)\} \subset \Gamma(\epsilon_n)$ is an approximating sequence for(EP_N), it follows the generalized well-posedness of (EP_N) that there exists a subsequence converge to some point of $(t, x) \in S$, which contradicts (3.13).
Conversely, suppose that (3.12) holds. By Lemma 3.3 and 3.4. $\Gamma(\epsilon)$ is a nonempty and closed for all $\epsilon > 0$. By the Kuratowsky Theorem [13], one can obtain

where $S = \bigcap_{\epsilon>0} \Gamma(\epsilon)$ is a nonempty and compact. Let $\{(t_n, x_n)\} \subset K$ be any approximate solution sequence for (EP_N). So there exists a nonnegative sequence $\{\epsilon_n\}$ with $\{\epsilon_n\} \to 0$ as $n \to 0$ such that

This means that $(t_n, x_n) \in \Gamma(\epsilon_n)$. This together with (3.12) indicates that

Since S is compact, it follows that there exists $(\overline{t_n}, \overline{x_n}) \in S$ in which

Again, by the compactness of the solution set S, the sequence $(\overline{t_n}, \overline{x_n})$ has a subsequence $\{(\overline{t_{n_k}}, \overline{x_{n_k}})\}$ converge strongly to $\{(\overline{t_n}, \overline{x_n})\} \in S$. Therefore, the corresponding $\{(t_{n_k}, x_{n_k})\}$ subsequence of $\{(t_n, x_n)\}$ converge strongly to $\{(\overline{t_n}, \overline{x_n})\}$. Hence, (EP_N) is well-posed in the generalized sense.

4. Well-posedness for optimization problems with generalization parametric equilibrium constraints

In this section, we introduce the formulation of optimization problems with equilibrium constraint. Further, we present some well-posedness results for optimization problem.

Definition 4.1. A sequence $\{(t_n, u_n)\} \subset T \times K$ is said to be an approximating sequence for (OPNPEC) if

(i) there exists a nonnegative sequence $\{\epsilon_n\}$ with $\{\epsilon_n\} \to 0$ as $n \to \infty$ such that

(ii)

Definition 4.2. (OPNPEC) is said to be strongly well-posed (resp., strongly well-posed in the generalized sense) if (OPNPEC) has a unique solution x and for every approximating sequence for (OPNPEC) converge strongly to the unique solution (resp., if $S \neq \emptyset$ and every approximate solution sequence has a subsequence which strongly converges to some point of S).
The set of approximating solutions of (OPNPEC) is defined by

Theorem 4.3. Assume that K is a nonempty convex, closed subset of a Banach space X. Let $F : T \times K \times K \longrightarrow \mathbb{R}$, $h : T \times K \longrightarrow \mathbb{R}$ and $\Psi, \alpha : K \times K \longrightarrow \mathbb{R}$ be four functions. If (OPNPEC) is strongly well-posed, then

Moreover, if the following assumptions hold:

(i) $x \longmapsto \langle Ax, y - x\rangle$ is u.s.c on K with respect to weak* $-$ topology of X^*,
(ii) $F(t, x, x) = 0$, $\forall t \in T, x \in K$,
(iii) $F(., x, .)$ is l. s. c and convex $\forall$ $x \in K$,
(iv) $F(t, ., .)$ is $\alpha -$ monotone bifunction, hemicontinuous, $\forall t \in T$,
(v) $\alpha(., y)$ and $\Psi(., y)$ are u. s. c $\forall y \in K$,
(vi) $\Psi(x, .)$ and $\alpha(x, .)$ are convex $\forall x \in K$,
(vii) $\Psi(., y)$ is u. s. c $\forall y \in K$,
(viii) h is l.s.c.
Then the converse holds.
Proof. Assume that (OPNPEC) is strongly well-posed. Then (OPNPEC) admits a unique solution $(t, x) \in T \times K$, i. e.,

Obviously, $\eta(\epsilon, \delta) \neq \emptyset$ for any $\epsilon, \delta > 0$, since $(t, x) \in \eta(\epsilon, \delta)$ for any $\epsilon, \delta > 0$. If daim $\eta(\epsilon, \delta) \nrightarrow 0$ as $\epsilon \to 0, \delta \to 0$ then there exists a constant $p > 0$ and $\{\epsilon_n\}$, $\{\delta_n\}$ with $\{\epsilon_n\} \to 0$, $\{\delta_n\} \to 0$ and $(t_n, x_n), (t_n, y_n) \in \eta(\epsilon_n, \delta_n)$ in which

Since, $(t_n, x_n), (t_n, y_n) \in \eta(\epsilon_n, \delta_n)$ $\forall n \in \mathbb{N}$, so both $\{(t_n, x_n)\}$ and $\{(t_n, y_n)\}$ are approximating sequence for (OPNPEC). By the well-posedness of (OPNPEC), they have to converge strongly to the unique solution of (OPNPEC) a contradiction to (4.2).

Conversely, suppose that condition (4.1) holds. Let $\{(t_n, x_n)\}$ be approximating sequence for (OPNPEC). So there exists a nonnegative sequence $\{\epsilon_n\}$ with $\{\epsilon_n\} \to 0$ in which

This yields that $(t_n, x_n) \in \eta(\epsilon_n, \delta_n)$. It follows from (4.1) that $\{(t_n, x_n)\}$ is a Cauchy sequence and so it converges sequence to a point $(t, x) \in T \times K$. It follows from 4.3 and assumptions $(ii - vi)$ that

Also, one can note from (4.3) and assumption (viii) that

$$\geq \lim_n \inf \; h(t_n, x_n)$$

So, by Theorem (2.6) (t, x) solve (OPNPEC). The uniqueness follows immediately from (4.1). Therefore, we complete the proof.

By the similar proof as that of Theorem 3.7, one can obtain the following result for the well-posedness of (OPNPE).

Theorem 4.4. Assume that T and K are nonempty, closed and convex subsets of real reflexive Banach spaces E and X respectively, if (OPNPEC) strongly well-posed in the generalized sense, then

$$\forall \epsilon, \delta > 0, \lim_{(\epsilon,\delta)\to(0,0)} \beta\left(\eta(\epsilon, \delta)\right) = 0, \text{where } \eta(\epsilon, \delta) \neq \emptyset.$$

Moreover, if the following assumptions hold:
(i) $x \longmapsto \langle Ax, y - x\rangle$ is u.s.c on K with respect to weak* $-$ topology of X^*,
(ii) $F(t, x, x) = 0$, $\forall t \in T, x \in K$,
(iii) $F\,(., x, .)$ is l. s. c and convex $\forall$ $x \in K$,

(iv) $F(t,.,.)$ is $\alpha -$ monotone bifunction, hemicontinuous, $\forall t \in T$,
(v) $\alpha(.,y)$ and $\Psi(.,y)$ are u.s.c $\forall y \in K$,
(vi) $\Psi(x,.)$ and $\alpha(x,.)$ are convex $\forall x \in K$,
(vii) $\Psi(.,y)$ is u.s.c $\forall y \in K$,
(viii) h is l.s.c.
Then the converse holds.

5. Conclusions

In this work, we introduce some concepts of well-posedness for a class of equilibrium problems with perturbations, which includes as a special case the class of equilibrium problems in [7, 8]. We establish some well-posedness results for a nonstandard equilibrium problem (EP_N) and for optimization problem involving α-monotone bifunction. Moreover, notice that several problems of well-posedness has been generalized to nonconvex variational inequalities, saddle point problems, fixed point problems, mathematical programming, Nash equilibrium problems, optimization problems with variational inequalities constrains, optimization problems with Nash equilibrium constrains.

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